Quantum process tomography as a tool for analyzing an ion trap quantum computer

Mark Riebe[1], C. Roos[1,2], H. Häffner[1,2], W. Hänsel[1], T. Körber[1], K. Kim[1], D. Chek-Al-Kar[1], M. Chwalla[1], J. Benhelm[2] & R. Blatt[1,2]

[1]Institut für Experimentalphysik, Universität Innsbruck, Technikerstraße 25, A-6020 Innsbruck, Austria
[2]Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften, Austria

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Introduction:

Quantum process tomography is a procedure to determine the linear map $\mathcal{E}$(r), which completely characterizes a quantum process. Knowledge of E(r) allows to assess the performance of quantum gates and quantum algorithms.

Theory:

In our experiments we work with quantum systems, which interact with a noisy environment. For these open quantum systems the action of process is described by a *completely positive map* $\mathcal{E}($. This map can be written in the *operator-sum representation* as:

$$\mathcal{E}(\rho) = \sum_i E_i \rho E_i^\dagger$$

The operators E_i can be expressed using a fixed set of basis operators A_m: $E_i = \sum_m e_{im} A_m$

Rewriting the positive map $\mathcal{E}$ we obtain: $\mathcal{E}(\rho) = \sum_{mn} \chi_{mn} A_m \rho A_n^\dagger$ with $\chi_{mn} = \sum_i e_{im} e_{in}^*$

This is known as the *chi-matrix representation*. By measuring the output density matrices $_{out} = \mathcal{E}(\ _{in}$ of a set of linear independent input states $_{in}$, the transfer matrix can be obtained by inverting the relation given above.

Reconstruction of the positive map $\mathcal{E}($:

1. Measurements: For at least 4^N input states $_{in,i}$ (N: number of qubits) a quantum state tomography of the output states $\mathcal{E}(\ _{in,i})$ is carried out.

2. Maximum likelihood reconstruction: The obtained tomographic results are evaluated by an iterative optimization routine, which gives the maximum likelihood estimate of the transfer matrix.
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3. Standard basis of operators: We use a basis of operators which are products of the 1-qubit Pauli operators:

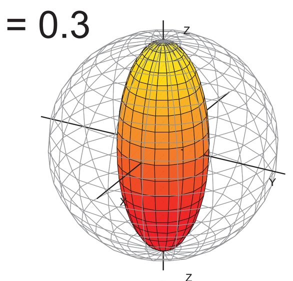
$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Y' = -i \cdot Y = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Identity — Bit flip — Bit + phase flip — Phase flip

Simple examples:

1) Phase flip:

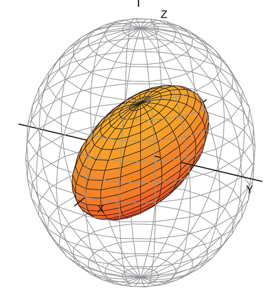
$$\mathcal{E}(\rho_{in}) = p \cdot \rho_{in} + (1-p) \cdot Z \rho_{in} Z$$



Block sphere is contracted in x-y plane

2) Bit flip:

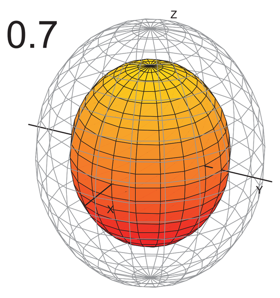
$$\mathcal{E}(\rho_{in}) = p \cdot \rho_{in} + (1-p) \cdot X \rho_{in} X$$



Bloch sphere is contracted in y-z plane

3) Depolarizing channel:

$$\mathcal{E}(\rho_{in}) = p \cdot \rho_{in} + (1-p) \cdot I/2$$



whole Bloch sphere is contracted

4) Measurement in Z-direction:

$$\mathcal{E}(\rho_{in}) = \langle 0|\rho_{in}|0\rangle \cdot |0\rangle\langle 0| + \langle 1|\rho_{in}|1\rangle \cdot |1\rangle\langle 1| = (I\rho_{in}I + Z\rho_{in}Z)/2$$

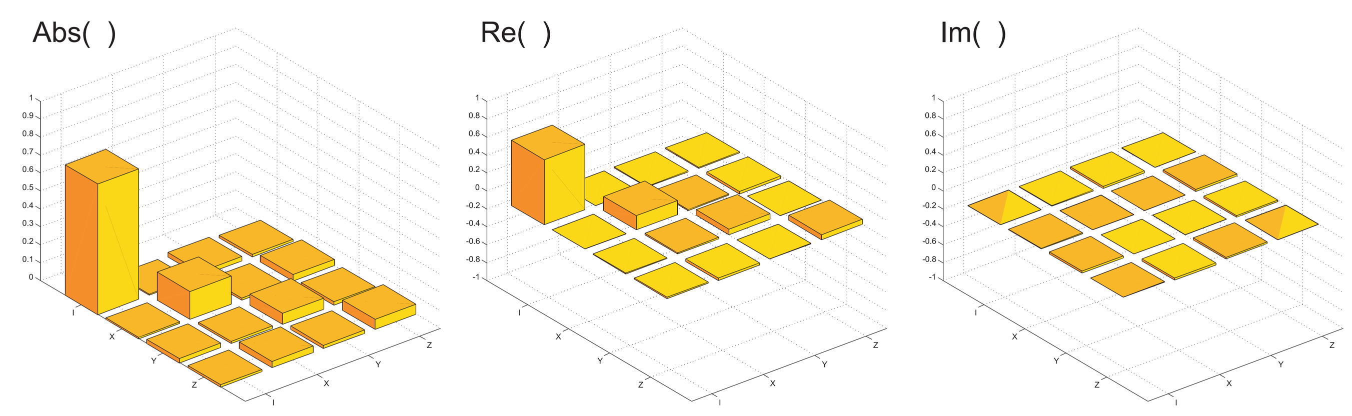
Bloch sphere is contracted to line along z-axis....

Applicaton 1: Quantum Teleportation

Within a three-ion crystal the quantum information stored in ion 1 is transferred to ion 3 using a quantum teleportation algorithm [1]. For ideal teleportation the output state of ion 3 would be the same as the input state of ion 1, i.e. the expected quantum process would be the identity.

[1] M. Riebe et. al, *Deterministic quantum teleportation with atoms* , Nature 429, 734 - 737 (17 Jun 2004)

Reconstructed transfer matrix :
Reconstruction from tomographic data of four input states:



Analysis:

1) Fidelity:
In order to compare the measured quantum process with the ideal evolution (Identity!) we use two different fidelity measures. The process fidelity results from comparing the two transfer matrices and is given by:

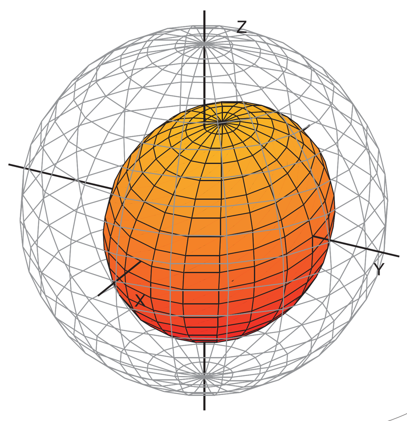
$$F_{proc} = tr(\chi_{id} \cdot \chi_{exp}) = 75\%$$

By using the experimentally obtained , we calculated the output density matrices for a large number of random input states. A comparison to the ideal output states yields the mean fidelity:

$$\bar{F} = 81\%$$

2) Action on Bloch sphere:

Bloch sphere is contracted, indicating the mixture of the ouput states. Additionaly the Bloch sphere is slightly tilted and shifted by the quantum process.



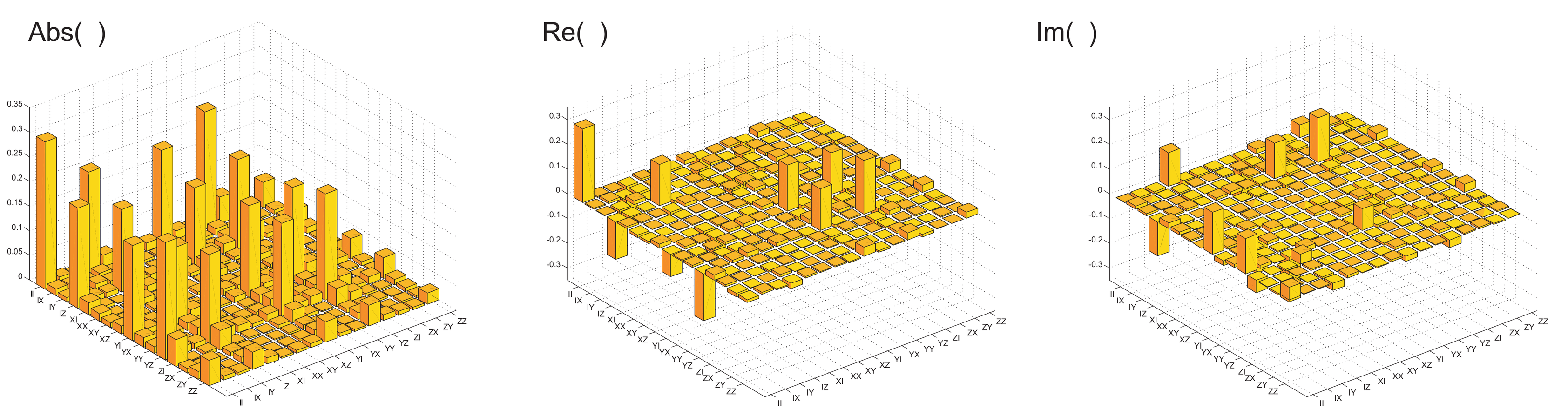
Note that the features of the transfer matrix and its action on the Bloch sphere rule out that Alice measured her qubit and transmitted the result to Bob, which would yield a 66 % mean fidelity.

Application 2: Cirac-Zoller controlled NOT gate

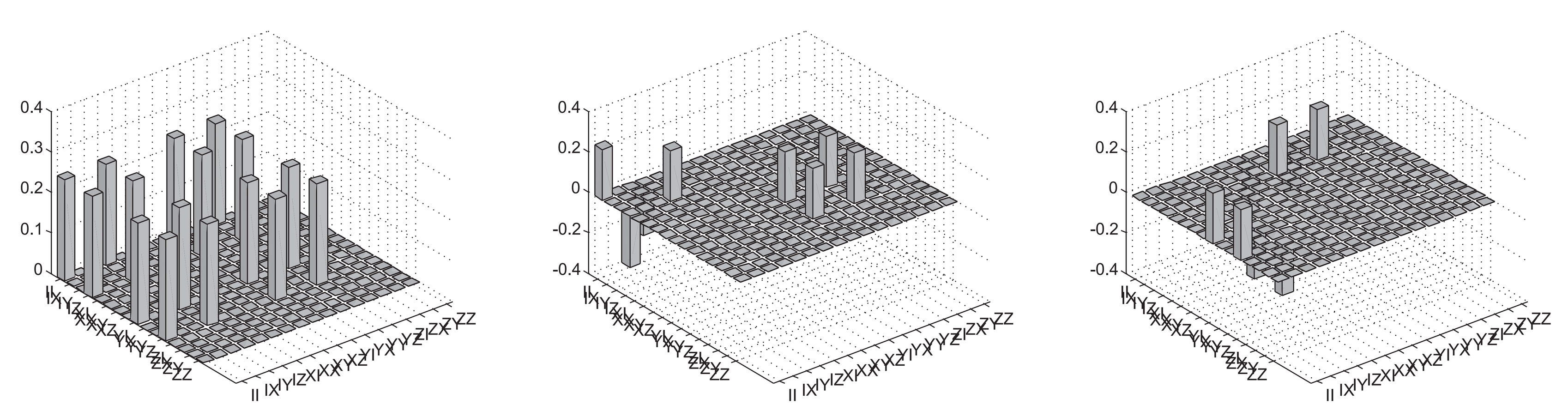
The investigated quantum process was a Cirac-Zoller controlled NOT gate between two ions in a linear ion trap. The ideal unitary evolution is given by:

$$U_{CNOT} = -\tfrac{1}{2}\left(I \otimes I - I \otimes Z + iY \otimes + iY \otimes Z\right)$$

Reconstructed transfer matrix
From the tomographic mesurements of 16 input states (tot. meas. 144) we obtain the transfer matrix:



Ideal transfer matrices:



Analysis:

1) Fidelity: The process fidelity between the measured transfer matrix $_{exp}$ and the the ideal trransfer matrix $_{id}$ is given by:

$$F_{proc} = tr(\chi_{id} \cdot \chi_{exp}) = 70\%$$

The mean fidelity over a set of 25 000 random input states is given by:

$$\bar{F} = 76\%$$

2)Entanglement capability + Mixedness:
For a set of random input states (25k states) we calculated the output state and calculated the change in entanglement (Tangle) and the mixedness of the output state (norm. linear entropy).